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LITIGATION OF SETTLEMENT DEMANDS QUESTIONED
BY BAYESIAN DEFENDANTS

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ABSTRACT

This paper analyzes a stylized model of pretrial settlement negotiations in a personal-injury case. It is assumed that the prospective plaintiff knows the severity of his injury but that the prospective defendant has incomplete information. As a result of this information asymmetry a proportion of slightly-injured plaintiffs are tempted to inflate their settlement demands and a proportion of such demands are rejected by suspicious defendants. By analogy with other models of adverse selection (e.g., Rothschild-Stiglitz [1976]), the presence of slightly-injured plaintiffs imposes a negative externality on plaintiffs with genuine severe injuries since defendants cannot identify the severely-injured and sometimes reject their reasonable demands, forcing them into costly litigation. A filing fee imposed on minor claims is shown to displace the equilibrium but, paradoxically, to cause an increase in the frequency of litigation.

This model differs from recent contributions to the literature on pretrial negotiations under incomplete information. Unlike P'ng (1983) and Bebchuk (1983), the uninformed litigant in this model learns from the observed equilibrium behavior of the informed litigant. Unlike Ordover-Rubinstein (1983) and Salant-Rest (1982), settlement demands are endogenous.

I. Introduction

It is often argued that if some bargain exists that all participants strictly prefer to a specified outcome, then surely that specified outcome will not occur. Coase (1960) makes such an argument in his analysis of bargaining over externalities. Gould (1973) and Shavell (1982) invoke the same line of reasoning in their analyses of pre-trial settlements. And yet there are many occasions when outcomes do occur that are Pareto inefficient--or at least that are revealed to be, ex post. For example peace (resp. labor) negotiations collapse and wars (resp. strikes) ensue which each side subsequently recognizes to be more costly than some settlement would have been. Trading on a futures contract terminates without full offsets and the shorts must incur transport costs to deliver physical goods--and the longs must likewise incur costs to reship these goods to the cash market--when both sides would have been better off with some paper settlement prior to the close of futures trading. As Arrow (1979, p. 25) has pointed out in discussing the Coase theorem, the argument that rational bargainers will always reach some Pareto efficient agreement depends on the implicit--and often unrealistic--assumption that they have complete information. If, instead, information is private, a negotiator may well reject a Pareto-dominant proposal because a rational inference from his limited information leads him to infer that some alternative--which is in fact Pareto-dominated--is the more attractive option. Situations where it is common knowledge that private information exists are characterized on the one hand by bluffing, deception, and concealment and on the other by discovery, challenging, and attempted inference. In such circumstances, an offer by one side or a response by the other often becomes the basis for a revision of beliefs about the strength of the opponent's position.

Below a legal conflict is examined in which self-interested adversaries have incomplete information. Agents in the model are assumed to be rational but not clairvoyant. In particular, an accident occurs and the severity of the prospective plaintiff's injury is not known with certainty to the prospective defendant. It is assumed that--for each possible severity of injury--there exists an out-of-court settlement which both parties would prefer to trial. But since each prospective plaintiff knows that his adversary has incomplete information about the extent of his injuries, in equilibrium some plaintiffs find it in their interest to inflate their claims--that is, to demand more than any defendant would be willing to pay if he had complete information about their injuries. Prospective defendants are, of course, painfully aware of their limited information and know that some plaintiffs exploit defendants' ignorance by making inflated claims. As a result, in equilibrium a defendant facing a high demand may well reject it in the rational (albeit possibly incorrect) expectation that the particular plaintiff he is challenging is making an inflated claim. The probability that a case goes to trial is determined endogenously in our model.

Representation of the legal settlement process as a noncooperative game of incomplete information helps clarify why out-of-court settlements sometimes fail to occur. In addition, such theorizing has practical value. Periodically, proposals are made to alter legal procedures (discovery rules, filing fees, and so forth). For any such change, it is desirable to determine its consequences--before the "reform" is implemented. But such procedural changes sometimes have important effects on the behavior of the legal adversaries. Without a behavioral model of how legal adversaries interact, there is no way to make a sensible prediction about how behaviors will be affected when--as is often the case--the procedural change has no precedent. Models like the one developed here permit predictions

of the behavioral responses which would be induced by a proposed change in legal procedures. The predictions are based on the plausible (yet testable) premise that legal adversaries act as if in a self-interested manner.

The analysis below may usefully be distinguished from independent work by Bebchuk (1983) and P'ng (1983). Each considers legal negotiations as a game where learning can occur. Each builds upon earlier models which investigate the financial incentives to litigate, such as the models of Landes (1971), Gould (1973), Posner (1977), Landes and Posner (1979), Shavell (1982), and Cooter et al. (1982).

Bebchuk (1983) considers a game of incomplete information in which a plaintiff makes some real-valued settlement demand on a take-it-or-leave-it basis. The defendant then either settles or goes to trial. In these respects his model is identical to ours. However, in the application of interest to Bebchuk, the defendant--not the plaintiff--has the private information. Since it is the uninformed player who moves first, nothing can be learned from his settlement demand.

P'ng (1983) considers a game of incomplete information in which a plaintiff makes an exogenous settlement demand on a take-it-or-leave-it basis. The defendant either settles or rejects the demand. If he rejects it, the plaintiff has the opportunity to withdraw the suit or proceed to trial. P'ng, like Bebchuk, assumes that the defendant has private information. Since the plaintiff moves (for the second time) after the defendant, the plaintiff can in principle learn from the prior move of the informed player. However, because P'ng arbitrarily restricts his players to pure strategies, the resulting restricted equilibria involve pure pooling and the uninformed player cannot infer anything about the defendant's information by observing his prior move.

In contrast to either of these papers, inference and misinference, honesty and deception are central to our model. The paper proceeds as follows. In the next section, we formulate the legal-settlements game which will be analyzed and discuss the solution concept which will be used. In Section III, the model is solved and the equilibrium strategies are calculated in an intuitive manner. Section IV illustrates how the model can be used to predict the consequences of exogenous changes in legal procedures. In Section I through IV, it is assumed that plaintiffs are of two possible types (slightly-injured and severely-injured). Section V indicates aspects of the analysis which carry over to the n-type case.

II. Formulation of the Legal Settlements Game

We examine the following stylized situation. One person causes an automobile accident in which a second party is injured. Although both parties observe the circumstances of the accident, only the injured party knows accurately the extent of his own injuries. To be sure, the motorist who is at fault knows generally that--in accidents of this kind--there is some probability (p_1) that the injury to the other party is slight (s_1) and complementary probability (p_2) that the injury is severe (s_2). But he does not know which outcome actually occurred in this instance.

After a short time, the injured party files a settlement demand which the potential defendant can either accept or reject. If he accepts, he avoids the costs of mounting a legal defense in court. If he rejects the demand, the case goes to trial. The outcome of the trial is assumed to depend only on the circumstances of the accident and the true injury sustained--not on the magnitude of the rejected settlement demand.

If the case goes to trial and the plaintiff is in fact slightly-injured (resp. severely-injured), his net expected payoff from the trial--after deducting lawyer's costs--is W_1 (resp. $W_2 > W_1$). In such circumstances, the expected cost to the defendant--inclusive of legal fees--is V_1 (resp. $V_2 > V_1$).

If the case is settled out of court, the costs the defendant incurs depend only on the magnitude of the demand he accepts. If the plaintiff made a settlement demand which cost the defendant d --inclusive of lawyer's costs--then the net payoff to the plaintiff (after deducting his costs) is denoted $g(d)$, where $g'(d) > 0$ and $g(d) \leq d$.

W_i , V_i , p_i , and $g(\cdot)$ are exogenous. Given these exogenous factors, our goal is to characterize the settlements demanded by potential plaintiffs of each type and their acceptance or rejection by potential defendants. In characterizing these outcomes, we assume that the strategies selected by each player form a Bayesian Nash equilibrium. That is, each player's strategy is optimal given correct conjectures about the strategies of the other players and full use of his information. For the defendant faced with a particular settlement demand, this means that--in evaluating his expected cost of challenging a plaintiff of unknown type at trial--he uses Bayes' theorem to revise his prior beliefs concerning the severity of the plaintiff's injury in light of his knowledge (based on the given equilibrium strategies of each type of plaintiff) of the relative likelihood that each type of plaintiff would make the demand he observes.

Define $F_i(x)$ to be the probability that a plaintiff of type i makes a settlement demand which would cost the defendant less than or equal to x to accept. Let $\alpha(d)$ be the probability that the defendant would accept a demand costing him d . The three functions-- $F_1(x)$, $F_2(x)$, and $\alpha(d)$ --constitute the strategies of the players.

In a Nash equilibrium, a plaintiff of type i is assumed to take the defendant's strategy as given and to select $F_i(x)$ to maximize his expected reward. Hence, given $\alpha(\cdot)$, $F_i(x)$ must satisfy the following inequality:

$$\int_{x \geq 0} \left\{ \alpha(x)g(x) + (1 - \alpha(x))W_i \right\} dF_i(x) \geq \int_{x \geq 0} \left\{ \alpha(x)g(x) + (1 - \alpha(x))W_i \right\} d\hat{F}_i(x)$$

for $i = 1, 2$ where $\hat{F}_i(x)$ is any feasible strategy the plaintiff might choose.^{1/} The defendant is assumed to select his rule $\alpha(d)$ for responding to observed demands so as to minimize his expected costs (inclusive of legal fees). That is, $\alpha(d)$ must satisfy the following inequality given d , $F_i(x)$, and p_i ($i = 1, 2$):

$$\alpha(d)d + (1 - \alpha(d)) \sum_{i=1}^2 \Pr(s_i|d)V_i \leq \hat{\alpha}(d)d + (1 - \hat{\alpha}(d)) \sum_{i=1}^2 \Pr(s_i|d)V_i$$

where $\hat{\alpha}(d)$ is any feasible strategy the defendant might choose. $\Pr(s_i|d)$ is computed by revising p_i according to Bayes' theorem--given the defendant's observation of d and his knowledge of the strategies of each type of plaintiff ($F_i(x)$).

Our formulation encompasses both pure and mixed strategies. For example, if a plaintiff of type i wished always to make a demand d , then this pure strategy could be represented as follows:

$$F_i(x) = \begin{cases} 0 & x < d \\ 1 & x \geq d. \end{cases}$$

Similarly, if the defendant wished always to accept some set of demands (A) and to reject every other demand, then this pure strategy could be represented as follows:

$$\alpha(d) = \begin{cases} 0 & d \notin A \\ 1 & d \in A. \end{cases}$$

Pure strategies arise in equilibrium whenever a single strategy for each player stands out as superior given the strategies of the other players. Mixed strategies arise in equilibrium only when several strategies tie as best for some player given the strategies of the other players. In the latter situation, pursuit of self-interest leads every player with a set of tied best strategies to select from within the set; but self-interested behavior does not insure that every individual player will select the same strategy from this set. Indeed, equilibrium may require that specific proportions of such players select particular strategies from the set of tied alternatives.

Suppose, for example, that defendants faced with a demand costing d to settle determine--given $F_i(x)$ --that taking a plaintiff to court would likewise be expected to cost d . In the absence of any financial incentive either way, a particular defendant might well elect always to settle (or, alternatively, always to litigate). But it should hardly be surprising if--of the many defendants who are indifferent when faced with d --some would choose to accept it while others would choose to reject it. Our analysis suggests, however, that what is indeterminate at the individual level can be determined at the aggregate level.^{2/} For, unless a certain proportion of the indifferent individual defendants accepted demand d , financial incentives would lead plaintiffs to depart from the given behaviors. Equilibrium analysis thus leads us to predict the proportion of defendants faced with equally preferred alternatives who make a particular choice. Below, we interpret $\alpha(d)$ to mean that--of all the defendants faced with demand d -- α would accept it while $(1 - \alpha)$ would reject it.

Similarly, suppose plaintiffs of type i are indifferent among a set of alternative demands. In the absence of any financial incentive one way or the other, a particular plaintiff of type i might always favor one element in this set of tied alternatives. But it should not be surprising if--of

the many type- i plaintiffs who are indifferent--some would make one demand while some would make another. However, what is indeterminate at the individual level can again be determined at the aggregate level. Unless a certain proportion of the indifferent type- i plaintiffs selected specific settlement demands, financial incentives would lead defendants to depart from the given behavior. Equilibrium analysis thus leads us to predict the proportion of type i plaintiffs who would make a particular demand when faced with equally attractive alternatives. Below, we interpret $F_i(x)$ to mean that--of all type- i plaintiffs in the population-- F_i would make demands costing less than (or equal to) x while $1 - F_i$ would make demands costing more than x .

The equilibrium problem we have posed may appear formidable. However, the reader will soon learn that its solution is straightforward. In the next section, we first characterize the broad features of the equilibrium in four intuitive propositions. We then solve explicitly for the equilibrium strategy of each player.

III. Determining the Equilibrium Strategies of the Game

We begin by establishing some of the properties of any equilibrium of our settlements game. We assume throughout that if the defendant had complete information about the plaintiff's type, then these adversaries could always reach a settlement which both would strictly prefer to trial. More formally, it is assumed that $g(V_i) > W_i$ for $i = 1, 2$. At trial, against a type- i plaintiff, the defendant would pay V_i and the plaintiff would receive W_i . If, instead, a settlement was reached which again cost the defendant V_i , the defendant would be equally well off and--by the foregoing assumption--the plaintiff would be strictly better off. If a slightly lower settlement were reached, both sides would be strictly better off than at trial. Under this assumption, trials would never occur if defendants had complete information about the severity of the injury of the prospective plaintiff. Instead a settlement preferred to trial by both parties would result. If defendants

have incomplete information, we will show that settlements sometimes fail to occur and the adversaries instead sometimes proceed to trial.

For simplicity, we assume that defendants automatically reject demands exceeding V_2 since settling such high demands costs more than going to trial no matter how severely the plaintiff is injured. The disposition of demands no larger than V_2 will of course depend on the strategy adopted in equilibrium by defendants.

We first state and prove four propositions which partially characterize the equilibria of our model:

Proposition 1: There is zero probability that settlement demands of less than V_1 will be made by either plaintiff.

Proof: Suppose, on the contrary, that in equilibrium some plaintiff made settlement demands with positive probability which are strictly less than V_1 . Faced with any such demand, a defendant would always accept it despite his uncertainty about the type of plaintiff he faces since in either case he would be worse off making a court challenge [if $d < V_1$, then $d < \min(V_1, V_2)$; hence $\alpha(d) = 1$]. Under this circumstance, any plaintiff making such demands with positive probability is not behaving optimally since he can increase his expected payoff by instead focussing that same probability on settlement demands marginally closer to V_1 .

Proposition 2: Demands exceeding V_2 will never be made by a slightly-injured plaintiff with positive probability and will only be made by a severely-injured plaintiff with positive probability if there is zero probability of reaching a settlement he would find more rewarding than trial.

Proof: By assumption, all demands exceeding V_2 are automatically rejected. Hence, a slightly-injured plaintiff would be better off instead demanding

V_1 with the same probability (since $g(V_1) > W_1$). A severely-injured plaintiff would demand more than V_2 with positive probability only if $\alpha(d) = 0$ for $g^{-1}(W_2) < d \leq V_2$; if there were any probability a demand in this interval would be accepted, the severely-injured plaintiff would prefer taking this chance to the certainty of rejection that would accompany a demand exceeding V_2 .

Proposition 3: Whenever the slightly-injured plaintiff makes two distinct settlement demands with positive probability in equilibrium, the severely-injured plaintiff must strictly prefer the higher demand. Whenever the severely-injured plaintiff makes two distinct settlement demands with positive probability in equilibrium, the slightly-injured plaintiff must strictly prefer the lower demand (provided the higher demand is accepted with positive probability). Consequently, none of the following situations can occur in equilibrium:

- (a) Each plaintiff assigns positive probability to the same n (>1) demand levels;
- (b) Both plaintiffs assign positive density to a common interval;
- (c) The slightly-injured plaintiff (resp. the severely-injured plaintiff) assigns positive density to some interval and the severely-injured plaintiff (resp. the slightly-injured plaintiff) assigns probability mass to any point in that interval except its upper boundary (resp. its lower boundary).

Proof: If the slightly-injured plaintiff makes two distinct demands with positive probability in equilibrium, each must be accepted with positive probability; otherwise the plaintiff would strictly prefer settlement at V_1 (since $g(V_1) > W_1$). Moreover, the higher demand must be accepted with

strictly smaller probability than the lower demand; otherwise he would never make the lower demand. For the slightly-injured plaintiff to make both demands with positive probability, he must in addition expect the same payoff from either choice:

$$\alpha(d_l)g(d_l) + (1 - \alpha(d_l))W_1 = \alpha(d_h)g(d_h) + (1 - \alpha(d_h))W_1$$

where d_l and d_h denote, respectively, the lower and higher demands.

Since $\alpha(d_l) > \alpha(d_h)$ and $W_2 > W_1$, it follows that

$$\alpha(d_l)g(d_l) + (1 - \alpha(d_l))W_2 < \alpha(d_h)g(d_h) + (1 - \alpha(d_h))W_2.$$

Hence, the severely-injured plaintiff would strictly prefer the higher demand as was asserted in the first half of Proposition 3.

The second half of Proposition 3 can be proved by a parallel argument showing that if the two demands result in equal expected payoffs to the severely-injured plaintiff then--provided $\alpha(d_l) > \alpha(d_h)$ --the expected payoff to the slightly-injured plaintiff from making the lower demand must be larger. The parenthetical qualification in the statement of the second half of the proposition is necessary to cover cases where the severely-injured plaintiff makes two demands with positive probability which are both always rejected ($\alpha(d_l) = \alpha(d_h) = 0$); in such cases, each plaintiff would be indifferent between the two demands.

To establish consequence (a), note that if each plaintiff made the same pair of demands with positive probability then the severely-injured plaintiff would have to be indifferent between two demands which yield equal expected payoffs to the slightly-injured plaintiff. But, as we have shown, the severely-injured plaintiff would always strictly prefer the higher demand. To establish consequences (b) and (c), first note that any plaintiff who assigns positive density to an

interval would obtain an unchanged expected payoff if instead he divided the same probability mass between discrete points in that interval. This fact can be used to "sweep" density assigned to an interval into mass points at levels suitably chosen so that Proposition 3 can be applied.^{3/}

Proposition 4: The strategies of the two types of plaintiffs must never permit a defendant who observes a settlement demand in (V_1, V_2) and who knows the equilibrium strategies to infer with probability one which type of plaintiff is making the observed demand. Consequently, neither of the following situations can occur in equilibrium:

- (a) One plaintiff assigns positive mass to a demand in (V_1, V_2) while the other plaintiff assigns no mass to that demand.
- (b) One plaintiff assigns positive density to an interval (containing values in (V_1, V_2)) while the other plaintiff assigns zero density to that interval.

Proof: Suppose, given the strategies of each type of plaintiff and confronted by a demand above V_1 , that the defendant could infer with probability one that the plaintiff is slightly injured. Such an inference would be valid, for example, if the slightly-injured plaintiff were the only plaintiff who made the observed demand with positive probability or, alternatively, if he alone assigned positive density to an interval containing the observed demand. Given the defendant's inference, he would always reject the observed demand. But since the slightly-injured plaintiff strictly prefers settlement at V_1 to trial ($g(V_1) > W_1$), the hypothesized strategy would not then be optimal for the slightly-injured plaintiff. Consequently, such a strategy would not occur in equilibrium.

Suppose, given the strategies of each type of plaintiff and confronted by a demand in (V_1, V_2) , that the defendant could infer with probability

one that the plaintiff is severely-injured. Such an inference would be valid, for example, if the severely-injured plaintiff were the only plaintiff who made the observed demand with positive probability or, alternatively, if he alone assigned positive density to an interval containing the observed demand. Given his inference, the defendant would always accept the observed demand. It follows that the slightly-injured plaintiff would never make a lower demand. Assuming that the defendant's inference is valid, we can conclude that the slightly-injured plaintiff must assign probability one to demands higher than the one observed. In particular, if the severely-injured plaintiff assigned positive density to an interval containing the observed demand, the slightly-injured plaintiff would have to assign probability one to demands at or above the upper boundary of this interval. If, alternatively, the severely-injured plaintiff assigned positive mass to the observed demand, the slightly-injured plaintiff would have to make higher demands with probability one. The arguments used to establish Proposition 3, however, rule out either of these possibilities.^{4/} Hence, the hypothesized situation cannot occur in equilibrium.

These four propositions severely restrict the combinations of strategies $(\alpha(\cdot), F_1(\cdot), F_2(\cdot))$ which need to be examined to determine the set of Nash equilibria. Proposition 1 implies that F_1 and F_2 are zero for $d < v_1$. Propositions 3 and 4 imply that neither F_1 nor F_2 can have positive slope for any $d \in (v_1, v_2)$; in that interval, both distribution functions are step-functions. Furthermore, Propositions 3 and 4 imply that each distribution function can have at most one jump in (v_1, v_2) ; furthermore, they imply that if one distribution function jumps somewhere in (v_1, v_2) , the other must jump at the same point. Finally, Proposition 2 implies that $F_1(v_2) = 1$ and that either $F_2(v_2) = 1$ or $\alpha(d) = 0$ for $d > g^{-1}(w_2)$. There remain ten qualitatively distinct types of situations which might occur as

Nash equilibria. Figure 1 distinguishes these equilibria by the location of the jumps in each distribution function. A jump in the i^{th} distribution function at point d means that the i^{th} plaintiff makes demand d with positive probability.

[Figure 1 goes here.]

o Elimination of Implausible Nash Equilibria Supported by Dominated Strategies

The requirement that the strategies selected by players in a game form a Nash equilibrium is weak and, unfortunately, it does not rule out all implausible combinations of strategies. To eliminate the unsatisfactory equilibria which remain, various refinements of the Nash solution have been suggested. Nash equilibria supported by dominated strategies seem particularly implausible. Hence, various refinements which have been proposed (trembling-hand perfectness, properness, and so forth) exclude such equilibria.

A strategy of player i is said to be "dominated" if player i has available an alternative strategy which is never inferior--no matter what the other players do--and is superior against some combination(s) of strategies which the other players have available. The argument that no player would ever choose a dominated strategy seems compelling. Choosing a dominated rather than an undominated strategy involves taking unnecessary risks--an increase in the payoff is never achieved and a strict decrease in the payoff can occur if the other players happen to select particular combinations of strategies available to them. The Nash restriction by itself does not eliminate such situations as long as--in the proposed equilibrium--the other players never choose any of the combinations of strategies which cause player i 's

dominated strategy to yield a payoff which is strictly lower than his alternative strategy would yield. Below, we follow the widespread practice of strengthening the Nash restriction by adding the restriction that the strategies the players select be undominated. This additional restriction suffices to eliminate cases d-i.

Consider first cases f-h. In each case, severely-injured plaintiffs sometimes make demands which are so high (exceeding V_2) that their rejection is inevitable. Such a strategy is dominated by a strategy of always making demands in the interval $(g^{-1}(w_2), V_2]$. Instead of insuring a trial, the latter strategy provides the same payoff if the demand is rejected and a higher payoff if the demand is accepted.^{5/} Under such circumstances, it seems implausible that a severely-injured plaintiff would ever demand more than V_2 .

Consider next cases i and j. In each case, severely-injured plaintiffs sometimes make demands $(g^{-1}(w_2))$ which are so low that the settlement they would receive is equal to the payoff expected at trial. For this to occur in equilibrium, every demand strictly preferred to trial ($d > g^{-1}(w_2)$) must be rejected. However, any demand in $(g^{-1}(w_2), V_2]$ weakly dominates a demand of $g^{-1}(w_2)$ since the same payoff occurs if it is rejected and a higher payoff occurs if it is accepted.

Finally, consider cases d and e. In either case, severely-injured plaintiffs sometimes demand an amount (V_2) which--under complete information--would leave the defendant indifferent between settling and going to trial. For such cases to be equilibria, it is impossible for all demands costing V_2 to be accepted; otherwise, slightly-injured plaintiffs would never demand less. Hence, in principle, either some V_2 demands are accepted or all are rejected. But all such demands cannot be rejected in a Nash equilibrium since severely-injured plaintiffs are indifferent between

demanding V_2 and pooling with the slightly-injured plaintiff at a demand accepted with positive probability in the interval $(g^{-1}(w_2), V_2)$.^{6/} Hence in cases d and e, defendants must accept some V_2 demands. However, such a strategy is dominated by another strategy available to the defendant--the rejection of all V_2 demands. If the plaintiff making the V_2 demand is in fact severely-injured, then this alternative strategy results in the same cost to the defendant as an out-of-court settlement. If, however, the plaintiff is slightly injured, the alternative strategy results in a lower cost to the defendant. Under these circumstances, it seems implausible that a defendant would ever settle a demand costing V_2 .

The Nash equilibria in the remaining cases (a-c) can each be supported by undominated strategies. The separating equilibrium (case c) requires that every slightly-injured plaintiff demand V_1 , every severely-injured plaintiff demand V_2 , and every demand exceeding $g^{-1}(w_2)$ be rejected. It can be verified that--for any set of exogenous parameters--the separating equilibrium is Pareto-dominated by the other equilibria which remain. The three possible types of Nash equilibria in undominated strategies (a-c) can be classified as follows:

- Case a: "Pure pooling" equilibria, in which all slightly-injured plaintiffs make the same settlement demand as the severely-injured plaintiffs;
- Case b: "Semi-pooling" equilibria, in which some slightly-injured plaintiffs make the same settlement demand ($d_2 > V_1$) as all severely-injured plaintiffs while the remainder of the slightly-injured plaintiffs demand V_1 ; and
- Case c: "Separating" equilibrium, in which all slightly-injured plaintiffs demand V_1 and get it while all severely-injured plaintiffs demand V_2 and wind up in court.

We now fully characterize these equilibria. We begin with semi-pooling equilibria and then find the separating equilibrium and the pure pooling equilibria to be limiting cases. For any set of exogenous parameters, there will exist a continuum of equilibria parameterized by d_2 . As we will discuss later, the multiplicity of equilibria in our game arises because of the assumed range of settlements which both defendants and severely-injured plaintiffs would find preferable to trial given complete information.

o Characterizing Each Type of Equilibrium

In any semi-pooling equilibrium, slightly-injured plaintiffs must be indifferent between making a demand which costs V_1 and making a demand which costs d_2 ; moreover, each of these alternatives must be at least weakly preferred to any other real-valued demand. Similarly, in any semi-pooling equilibrium, severely-injured plaintiffs must at least weakly prefer to make a demand costing d_2 to any other real-valued demand. These conditions place restrictions on $\alpha(d)$.

Let $\alpha_1(d)$ be the acceptance function which would make a slightly-injured plaintiff indifferent between demanding V_1 and demanding any higher amount d . Let $\alpha_2(d)$ be the acceptance function which would make a severely-injured plaintiff indifferent between demanding d_2 --which a slightly-injured plaintiff ranks equal with V_1 --and demanding any other amount d . That is, $\alpha_1(d)$ solves $g(V_1) = \alpha_1(d)g(d) + (1 - \alpha_1(d))W_1$ and $\alpha_2(d)$ solves $\alpha_1(d_2)g(d_2) + (1 - \alpha_1(d_2))W_2 = \alpha_2(d)g(d) + (1 - \alpha_2(d))W_2$. Solving for $\alpha_1(d)$ and $\alpha_2(d)$, we obtain:

$$\alpha_1(d) = \frac{g(V_1) - W_1}{g(d) - W_1}$$

$$\text{and } \alpha_2(d) = \frac{\alpha_1(d_2)g(d_2) + (1 - \alpha_1(d_2))W_2 - W_2}{g(d) - W_2} = \alpha_1(d_2) \left(\frac{g(d_2) - W_2}{g(d) - W_2} \right).$$

It is easy to verify that:^{7/}

$$\text{i. } \alpha_1(V_1) = 1$$

$$\text{ii. } \alpha_1(d) \begin{cases} > \alpha_2(d) & \text{for } d > d_2 \\ = \alpha_2(d) & d = d_2 \\ < \alpha_2(d) & d < d_2 \end{cases}$$

and iii. $\alpha_i(d) > 0$ and $\alpha_i'(d) < 0$ for $d > g^{-1}(W_i)$ and $i = 1, 2$.

We plot $\alpha_1(d)$ and $\alpha_2(d)$ in Figure 2. Each function is positive and strictly decreasing. $\alpha_1(d)$ (resp. $\alpha_2(d)$) approaches zero asymptotically as d approaches infinity and approaches infinity asymptotically as d approaches $g^{-1}(W_1)$ (resp. $g^{-1}(W_2)$). The convex curvature of the functions plotted in the diagram is merely illustrative and will not be used in the analysis. It is useful to think of $\alpha_1(d)$ as an indifference curve indicating the locus of $\alpha - d$ combinations which the slightly-injured plaintiff would find as desirable as the $\alpha - d$ combination $(1, V_1)$. Similarly, $\alpha_2(d)$ can be regarded as an indifference curve indicating the locus of $\alpha - d$ combinations which the severely-injured plaintiff would find as desirable as the $\alpha - d$ combination $(\alpha_1(d_2), d_2)$.

Define $\alpha(d)$ as follows:

$$\alpha(d) \geq 0 \text{ for all } d, \text{ with equality at } d = V_2$$

$$\alpha(d) = 1 \text{ for } d \leq V_1, \text{ and}$$

$\alpha(d) \leq \min(\alpha_1(d), \alpha_2(d))$ with equality iff $d = V_1$ or $d = d_2$.

By assumption, $\alpha(d) = 0$ for $d > V_2$. Clearly, $\alpha(d) \in [0, 1]$ if $d_2 > g^{-1}(w_2)$.

Hence for d_2 in this region, $\alpha(d)$ is a feasible strategy for defendants.

We now verify that--if defendants adopt $\alpha(d)$ as their strategy--plaintiffs will find semi-pooling behavior to be optimal. Consider Figure 2. In the diagram, $\alpha(d)$ lies below the heavily shaded curve, $\min(\alpha_1(d), \alpha_2(d))$, touching it only twice--at $d = V_1$ and $d = d_2$. Given $\alpha(d)$, a slightly-injured plaintiff is indifferent between V_1 and d_2 and strictly prefers either demand to any other while the severely-injured plaintiff strictly prefers d_2 to any other demand. Hence, given $\alpha(d)$ either type of plaintiff would strictly reduce his payoff if he were to change his strategy unilaterally.^{8/}

[Figure 2 goes here.]

Next, consider the behavior of defendants. In any semi-pooling equilibrium, they must accept the common demand (d_2) with a probability which is positive but less than one. For if they always accepted such demands, no slightly-injured plaintiff would ever demand V_1 ; and if they never accepted d_2 demands, every slightly-injured plaintiff would prefer to make a demand costing V_1 .

Since in a semi-pooling equilibrium, every severely-injured plaintiff makes a demand costing less than V_2 , defendants would strictly prefer to accept all such demands were it not for the fact that some slightly-injured

plaintiffs also make them. Indeed, for defendants to be indifferent between settling at a cost of d_2 and challenging a plaintiff of unknown type at trial, there must be "just enough" slightly-injured plaintiffs inflating their claims to make a court challenge equally attractive as a defense tactic. Let $\pi_{1,2}$ denote the fraction of the slightly-injured plaintiffs who make a demand costing d_2 ; the complementary fraction makes a demand costing V_1 . If the defendant is indifferent between settling at a cost of d_2 and taking the plaintiff of unknown type to trial, then:

$$d_2 = \left(\frac{\pi_{1,2} p_1}{\pi_{1,2} p_1 + 1(1 - p_1)} \right) V_1 + \left(\frac{1(1 - p_1)}{\pi_{1,2} p_1 + 1(1 - p_1)} \right) V_2$$

where $\pi_{1,2} \in [0, 1]$.

This implies

$$\pi_{1,2} = \left(\frac{1 - p_1}{p_1} \right) \left(\frac{V_2 - d_2}{d_2 - V_1} \right) \leq 1.$$

If d_2 is the common demand in a semi-pooling equilibrium, it must satisfy two restrictions: i. d_2 must result in $\pi_{1,2} \in (0, 1)$ where $\pi_{1,2}$ is given by the foregoing formula; and ii. d_2 must be at least weakly preferred to trial by the severely-injured plaintiff. That is,

$$i. V_2 > d_2 > p_1 V_1 + p_2 V_2$$

and ii. $d_2 \geq g^{-1}(w_2)$.

Since $V_2 > p_1 V_1 + p_2 V_2$ and $V_2 > g^{-1}(w_2)$, there will always exist a continuum of semi-pooling equilibria. Equilibria with lower d_2 values must have higher $\pi_{1,2}$ values. To summarize our conclusions, we depict equilibrium strategies (α, F_1, F_2) which support the semi-pooling equilibrium

associated with a particular d_2 in the three panels of Figure 3.

[Figure 3 goes here.]

If $g^{-1}(w_2) > p_1 v_1 + p_2 v_2$, then no pooling equilibria exist and the semi-pooling equilibrium with the lowest common demand has $d_2 = g^{-1}(w_2)$. If, however, $g^{-1}(w_2) < p_1 v_1 + p_2 v_2$, then pooling equilibria do exist. As d_2 is reduced below v_2 , the region of semi-pooling equilibria is traversed. The lower is d_2 , the higher is $\pi_{1,2}$. When $d_2 = p_1 v_1 + p_2 v_2$, $\pi_{1,2} = 1$ and we enter the region of pure pooling. Throughout this region ($g^{-1}(w_2) \leq d_2 \leq p_1 v_1 + p_2 v_2$), d_2 is low enough that any common demand is acceptable to the defendant but d_2 is not so low that the severely-injured plaintiff would prefer trial to settlement. In a pure pooling equilibrium both plaintiffs demand d_2 and--although the defendant cannot learn anything since both plaintiffs always make the common demand--nevertheless the defendant determines that the expected cost of trial (computed using prior probabilities, p_i) exceeds the cost of settling (d_2). Consequently, he always settles.

We first characterize the strategy of the defendants which supports a pure-pooling equilibrium.

Let

$$\alpha(d) \geq 0 \text{ for all } d, \text{ with equality at } d = v_2$$

$$\alpha(d) = 1 \text{ for } d \leq d_2, \text{ and}$$

$$\alpha(d) < \frac{g(d_2) - w_2}{g(d) - w_2} \text{ for } d > d_2.$$

By assumption, $\alpha(d) = 0$ for $d > v_2$.

Since $\alpha(d) \in [0, 1]$ for $d_2 \geq g^{-1}(w_2)$, $\alpha(d)$ is a feasible strategy for defendants.

Note that given this strategy for defendants, both type of plaintiffs would strictly prefer a demand of d_2 to any other real-valued demand. Moreover, since $d_2 \leq p_1 v_1 + p_2 v_2$ the defendant would prefer to accept all such demands. To summarize this discussion, we depict the equilibrium strategies which support the pure pooling equilibrium associated with a particular d_2 in the three panels of Figure 4.

[Figure 4 goes here.]

As we have indicated, a region of semi-pooling equilibria always exists and a region of pure pooling equilibria may also exist. One other equilibrium also always exists--the separating equilibrium. It can be viewed as a limiting case of the semi-pooling equilibrium in which $d_2 = v_2$.

The following strategy for the defendants will support the separating equilibrium:

$$\alpha(d) \geq 0 \text{ for all } d$$

$$\alpha(d) = 1 \text{ for } d \leq v_1$$

$$\alpha(d) \leq \alpha_1(d) \text{ with equality iff } d = v_1 \text{ and}$$

$$\alpha(d) = 0 \text{ for } d \geq g^{-1}(w_2).$$

Given this strategy for defendants, a slightly-injured plaintiff will strictly prefer to demand v_1 and a severely-injured plaintiff will be indifferent among demands at least as great as $g^{-1}(w_2)$. Hence, the severely-injured plaintiff cannot improve upon a demand of v_2 . If slightly-injured plaintiffs always demand v_1 and severely-injured plaintiffs always demand v_2 , the defendant can do no better than to accept all the low demands and reject all the high demands. To summarize this discussion, we depict the equilibrium strategies which support the separating

equilibrium in the three panels of Figure 5.

[Figure 5 goes here.]

The model we have been discussing has a separating equilibrium, a continuum of semi-pooling equilibria, and may also have a continuum of pooling equilibria--each supported by undominated strategies.^{9/} It is natural to ask if any of these can be eliminated by the imposition of the reasonable, additional restriction that the supporting strategies be rational ("credible")--even in contingencies which occur with zero probability in equilibrium. In our model, the defendant's strategy describes how he will react to any real-valued demand even though at most two demands occur in equilibrium with positive probability. What would be a self-interested response for the defendant if he were confronted by some other demand? Kreps and Wilson would require that the defendant's conjectures about the plaintiff's type be made explicit and subjected to certain restrictions.^{10/} Selten has proposed stronger restrictions which can be adapted for games where the sets of pure strategies are not finite.^{11/} Both criteria are discussed in Cave [1984], who shows that neither restriction suffices to eliminate any of our equilibria.

It might also be thought that increasing the number of plaintiff-types would eliminate the multiplicity of equilibria. However, a continuum of equilibria exists for any finite number of plaintiff-types as well as for a continuum of such types.^{12/}

In concluding the section, therefore, it seems appropriate to note the source of this indeterminacy. In the pure pooling and semi-pooling equilibria, d_2 must be less than v_2 and greater than $g^{-1}(w_2)$. That is, d_2 must lie in the complete-information bargaining range for a game against a severely-injured plaintiff. It is the lack of other restrictions on d_2

which causes the continuum of equilibria.

What makes this indeterminacy striking is that settlement games where the first mover makes a take-it-or-leave-it demand have a unique equilibrium when played under different information structures.^{13/} For example, under complete information a plaintiff of type i would know that the defendant knows his identity and that the defendant would accept any demand up to v_i and no demand beyond it. No other $\alpha(d)$ function would be cost-minimizing for the defendant. Given this $\alpha(d)$ function, the plaintiff would demand v_i with probability one. Although no other demand would be made with positive probability, nonetheless the defendant would have to conjecture that any other demand encountered was made by the type- i plaintiff; hence he would accept lower and reject higher demands. Since the strategy of the defendant leads to plaintiff behavior which--when coupled with the defendant's conjectures--in turn rationalizes the strategy of the defendant, it supports a Nash equilibrium. Since only one strategy has this property, the equilibrium is unique. In our case, however, there exist many acceptance strategies for the defendant ($\alpha(d)$) which are rational given the plaintiff behavior (and conjectures about it) which they induce.

IV. Effects of Changing Legal Rules

To predict the consequences of a change in legal rules, we need to be able to forecast how legal adversaries will adapt to the new environment in which they find themselves. The difficulty of this task should not be underestimated. But failure to accomplish it will result in unrealistic expectations about the consequences of legal reforms. Brazil [1978], for example, contrasts what was expected from the discovery rules first introduced in 1938 in the Federal Rules of Civil Procedure to their actual

effects. He argues that designers of the rules failed to take account of how legal adversaries would adapt their tactics to take advantage of [his word is "abuse"] the new rules.

The model described above can be used to forecast the consequences of changes in legal rules and court policies. The predicted changes in behavior rules are based on the plausible assumption that the players will adapt their strategies in whatever way best serves their self-interest in the new environment. To illustrate how the policy analysis can be conducted, we will examine the consequences of a hypothetical change in the legal environment--the imposition of a fee imposed on suits of less than some specified amount. We will show how this legal reform would affect the equilibrium behavior of plaintiffs and defendants and consequently how it would affect the frequency of litigation.

The methodology employed in this section is comparative statics. That is, we investigate how the equilibria induced by one vector of exogenous parameters compare to the equilibria induced by a different vector of such parameters. Since each vector of exogenous parameters induces a set of equilibria parameterized by d_2 , we must compare two sets of equilibria. To facilitate this comparison, we summarize the results of the last section in the two panels of Figure 6. Panel a plots $\pi_{1,2}^e(d_2)$, the probability that a slightly-injured plaintiff will inflate his demand in the equilibrium associated with each d_2 (as d_2 varies). Analytically, $\pi_{1,2}^e(d_2)$ is defined as follows:

$$\begin{aligned} \pi_{1,2}^e(d_2) &= 1 && \text{For } d_2 \in [g^{-1}(w_2), p_1 v_1 + p_2 v_2], \\ &&& \text{the pooling region.}^{14/} \\ &= \left(\frac{1 - p_1}{p_1} \right) \left(\frac{v_2 - d_2}{d_2 - v_1} \right) && \text{for } d_2 \in (p_1 v_1 + p_2 v_2, v_2), \\ &&& \text{the semi-pooling region} \\ &= 0 && \text{for } d_2 = v_2, \text{ the separating region.} \end{aligned}$$

Panel b plots $\alpha^e(d_2)$, the probability that the high demand is accepted in the equilibrium associated with each d_2 . Analytically, $\alpha^e(d_2)$ is defined as follows:

$$\begin{aligned} \alpha^e(d_2) &= 1 && \text{for } d_2 \in [g^{-1}(w_2), p_1 v_1 + p_2 v_2] \\ &= \left[\frac{g(v_1) - w_1}{g(p_1 v_1 + p_2 v_2) - w_1}, 1 \right] && \text{For } d_2 = p_1 v_1 + p_2 v_2, \text{ the boundary} \\ &&& \text{between the pooling and separating} \\ &&& \text{regions} \\ &= \frac{g(v_1) - w_1}{g(d_2) - w_2} && \text{for } d_2 \in (p_1 v_1 + p_2 v_2, v_2) \\ &= 0 && \text{for } d_2 = v_2. \end{aligned}$$

In our comparative-static exercises we will show how each of these functions (the curves in Figure 6) shifts in response to the imposition of the filing fee.

[Figure 6 goes here.]

Our results can be interpreted as showing how the behavior of the defendants and plaintiffs would change in response to the imposition of the filing fee if d_2 did not change (or, more generally, if it changed in any specified way). Since d_2 can be observed, the predictions of the model are testable.

Suppose that in a state court system, cases where the plaintiff demands less than some minimal amount (denoted d_f) must be filed in a lower court, (i.e., a court of limited jurisdiction), while a plaintiff demanding more must file his case in a higher court (i.e., a court of general jurisdiction). Furthermore, let us assume that V_1 is below the cutoff, while d_2 is always above it. That is, $V_1 < d_f < g^{-1}(W_2) \leq d_2$. In other words, low demands must be filed in a lower court, while high demands must be filed in a higher court. Next, suppose that the lower court judge imposes a fee (f) which must be paid for filing a demand below d_f .^{15/} This fee must be paid whether the case is settled out of court or goes to trial. Hence the defendant's net costs are the same as they were previously. However, the plaintiff now receives $g(d) - f$ (for $d < d_f$) if any such low demand is settled out of court and $W_1 - f$ if he goes to trial. The plaintiff is assumed to receive the same payoffs as before in all other situations ($d \geq d_f$).

It is a simple matter to determine how legal adversaries would adapt their behavior given this exogenous change in the rules. The exogenous change does not alter the location of the boundary between the pooling and semi-pooling regions since it does not affect $p_1 V_1 + p_2 V_2$. Indeed, since the costs to the defendant (V_1 and V_2) remain unchanged, the curve $\pi_{1,2}^e(d_2)$ in Figure 6a does not shift. Finally, since defendants always accept d_2 demands in the pooling region and always reject them in the separating region, $a^e(d_2)$ will not shift in either of these two regions.

In the semi-pooling region, however, the equilibrium behavior of defendants ($a^e(d_2)$) must change in response to the altered cost-incentives of the plaintiffs. To determine the new behavior of defendants, we must re-derive $a(d)$ in the new policy regime.

In the presence of a filing fee, a slightly-injured plaintiff will be indifferent between a demand costing V_1 and any higher demand d if and only if the new acceptance function $\hat{a}_1(d)$ satisfies the following:

$$\begin{aligned} g(V_1) - f &= \hat{a}_1(d) [g(d) - f] + (1 - \hat{a}_1(d)) [W_1 - f] & \text{for } d < d_f \\ g(V_1) - f &= \hat{a}_1(d) [g(d)] + (1 - \hat{a}_1(d)) W_1 & \text{for } d \geq d_f . \end{aligned}$$

A severely-injured plaintiff will be indifferent between d_2 and higher demands if and only if the new acceptance function $\hat{a}_2(d)$ satisfies the following:

$$\hat{a}_1(d_2) g(d_2) + (1 - \hat{a}_1(d_2)) W_2 = \hat{a}_2(d) g(d) + (1 - \hat{a}_2(d)) W_2 \quad \text{for } d \geq d_2 .$$

Solving for $\hat{a}_1(d)$ and $\hat{a}_2(d)$, we obtain:

$$\hat{a}_1(d) = \begin{cases} \frac{g(V_1) - W_1}{g(d) - W_1} & \text{for } d < d_f \\ \frac{g(V_1) - W_1 - f}{g(d) - W_1} & \text{for } d \geq d_f \end{cases}$$

$$\text{and } \hat{a}_2(d) = \hat{a}_1(d_2) \left(\frac{g(d_2) - W_2}{g(d) - W_2} \right) .$$

Define $\hat{a}(d)$ as follows:

$\hat{\alpha}(d) \geq 0$ for all d , with equality at $d = v_2$

$\hat{\alpha}(d) = 1$ for $d < v_1$, and

$\hat{\alpha}(d) \leq \min(\hat{\alpha}_1(d), \hat{\alpha}_2(d))$ with equality iff $d = v_1$ or $d = d_2$.

By assumption, $\alpha(d) = 0$ for $d > v_2$.

$\hat{\alpha}_1(d)$, $\hat{\alpha}_2(d)$, and $\hat{\alpha}(d)$ are depicted in Figure 7. The $\hat{\alpha}(d)$ function supports a
[Figure 7 goes here.]

semi-pooling equilibrium in the presence of a fee on claims filed in lower court since a slightly-injured plaintiff is indifferent between v_1 and d_2 and strictly prefers either of these demands to any other while a severely-injured plaintiff strictly prefers d_2 to any other real-valued demand. In terms of panel b in Figure 6, $\alpha^e(d_2)$ shifts downward to $\frac{g(v_1) - w_1 - f}{g(d_2) - w_2}$ everywhere in the semi-pooling region. We illustrate how the behavior of defendants changes (across the set of equilibria) in Figure 8.

[Figure 8 goes here.]

Since $\hat{\alpha}^e(d_2) < \alpha^e(d_2)$ throughout the semi-pooling region, we conclude that as long as there was previously any litigation imposition of a fee on claims filed in lower court will increase the fraction of claims in higher court which go to trial.

The following argument explains the intuition behind this conclusion. Suppose in some semi-pooling equilibrium that the odds of having a high claim challenged did not increase. Then, every slightly-injured plaintiff previously indifferent as to the court in which to file would inflate his claim and would file in the higher court. This, of course, would not be an equilibrium because defendants would then want to increase their challenges to such claims. In equilibrium, the fraction of high demands which defendants challenge must increase by exactly enough that slightly-injured plaintiffs do not wish to inflate their claims any more than they did previously.

As a result, in equilibrium, the increase in the filing fee has no effect whatsoever on the fraction of the cases filed in the lower court or their disposition.

Note that the entire impact of the increase in the filing fee of the lower court spills over to the higher court. Our analysis suggests that when a lower court attempts to rid itself of minor cases ($d < d_f$) by increasing its filing fee, it may cause a higher proportion of cases in the higher court to be contested and taken to trial, thereby causing the higher court to suffer from a greater workload.

V. Extension of the Model to n Levels of Injury

Although space does not permit a complete discussion of the generalization to n levels of injury (s_i , $i = 1, \dots, n$), it seems important at least to indicate that the analysis and conclusions of the 2-type case can be easily extended. In what follows, we establish that in the n-type case, separating and semi-pooling equilibria always exist and pure pooling equilibria might or might not exist. Moreover, we show that our conclusion about the effect of the filing fee on a semi-pooling equilibrium carries over to the n-type case.

Let v_i denote the cost to the defendant of going to trial against a plaintiff with a type-i injury. Let w_i denote the net payment such a plaintiff expects from trial. Let d_i ($i = 1, \dots, n$) be n demand levels arrayed in increasing order ($d_1 < d_2 < d_3 < \dots < d_n$). Then we can construct a semi-pooling equilibrium as follows. Provisionally set $d_i = v_i$ for $i = 1, \dots, n$. Suppose the least seriously-injured plaintiff (s_1) demands either d_1 or d_2 , the next type (s_2) demands either d_2 or d_3 , ...

and the most seriously injured (s_n) always demands d_n . This pattern of settlement demands is depicted in Figure 9 and is the counterpart to panel b of Figure 1.

[Figure 9 goes here.]

Next we must find an $\alpha(d)$ function characterizing the defendant's behavior which induces each type of plaintiff to behave in the way we have depicted. Generalizing our previous approach, define $\alpha_i(d)$ as the locus of $\alpha - d$ combinations which makes a type- i plaintiff indifferent between a demand of d_i accepted with probability $\alpha_{i-1}(d_i)$ and a demand of d accepted with probability $\alpha_i(d)$. Then $\alpha_i(d)$ must solve the following recursive equation:

$$\alpha_{i-1}(d_i)g(d_i) + (1 - \alpha_{i-1}(d_i))w_i = \alpha_i(d)g(d) + (1 - \alpha_i(d))w_i$$

$$\Rightarrow \alpha_i(d) = \alpha_{i-1}(d_i) \left\{ \frac{g(d_i) - w_i}{g(d) - w_i} \right\} \text{ for } i = 1, \dots, n.$$

Setting $\alpha_0(d_1) = 1$, we can construct the sequence of functions $\{\alpha_i(d)\}$ recursively. They are portrayed in Figure 10, the counterpart to Figure 2.

[Figure 10 goes here.]

An $\alpha(d)$ function which makes the type- n (i.e., most severely-injured) plaintiff strictly prefer d_n and the i^{th} type ($i = 1, \dots, n-1$) strictly prefer either d_i or d_{i+1} to any other real-valued demand is easily constructed:

$$\alpha(d) \geq 0 \text{ for all } d, \text{ with equality at } d = V_n$$

$$\alpha(d) = 1 \text{ for } d \leq V_1 \text{ and}$$

$$\alpha(d) \leq \min(\alpha_1(d), \alpha_2(d), \dots, \alpha_n(d)) \text{ with equality iff } d = d_i,$$

$$i = 1, \dots, n.$$

We assume that $\alpha(d) = 0$ for $d > V_n$.

$\alpha(d)$ is also depicted in Figure 10.

Finally, in a semi-pooling equilibrium the relative proportions of the i^{th} and $i+1^{\text{st}}$ type at pool d_{i+1} (for $i = 1, \dots, n-1$) must leave the Bayesian defendant indifferent between settling and going to trial. For the defendant to be indifferent, the following equality must hold:

$$d_{i+1} = \Pr(s_i | d_{i+1})V_i + \Pr(s_{i+1} | d_{i+1})V_{i+1}.$$

Let p_i denote the proportion of type- i plaintiffs in the population and let $\pi_{i,j}$ be the probability in equilibrium that a type- i plaintiff makes a d_j demand. Then, from Bayes' theorem,

$$d_{i+1} = \frac{\pi_{i,i+1}P_iV_i + \pi_{i+1,i+1}P_{i+1}V_{i+1}}{\pi_{i,i+1}P_i + \pi_{i+1,i+1}P_{i+1}}$$

or

$$\pi_{i,i+1} = \frac{P_{i+1}}{P_i} \pi_{i+1,i+1} \left(\frac{V_{i+1} - d_{i+1}}{d_{i+1} - V_i} \right).$$

Since $\pi_{i,i} = 1 - \pi_{i,i+1}$ we conclude that

$$\pi_{i,i} = 1 - \frac{P_{i+1}}{P_i} \pi_{i+1,i+1} \left(\frac{V_{i+1} - d_{i+1}}{d_{i+1} - V_i} \right) \text{ for } i = 1, \dots, n-1.$$

If $d_i = V_i$ for $i = 2, \dots, n$, then $\pi_{i,i} = 1$. If d_i is set a little smaller, however, $\pi_{i,i} \in (0, 1)$. The result is one of a continuum of semi-pooling equilibria for the n -type case.

Consider the effect that a filing fee on demands below d_f would have

on this semi-pooling equilibrium. As before, we compare the original equilibrium with the corresponding new equilibrium having the same $\{d_i\}$ levels. Given the preceding formula, it is clear that once again the fee would not affect the equilibrium frequencies $(\pi_{i,j})$ with which plaintiffs of each type inflate their claims. But for plaintiff behavior not to change, defendants must become more litigious. Indeed, an interesting sort of chain-reaction must be set off.

Suppose the fee is charged for demands of less than d_f and $d_f \in (d_k, d_{k+1})$. The type- k plaintiff reasons that, ceteris paribus, he can avoid the filing fee by always demanding d_{k+1} and never demanding d_k . To restore his indifference, claims of d_{k+1} must be rejected more frequently. But this increase in court challenges gives the $k + 1^{\text{st}}$ type of plaintiff an incentive always to inflate his claims by demanding d_{k+2} . To restore his indifference, claims of d_{k+2} must be rejected more frequently... Clearly, litigation must increase at every demand level above d_k .

Separating equilibria also exist in the n -type case as they do in the 2-type case. To construct one, suppose plaintiffs of the lowest injury type demand V_1 and always get it while plaintiffs of other types each demand V_i ($i = 2, n$) and never get it. In this equilibrium the defendant rejects all demands exceeding $\min(g^{-1}(w_2), \dots, g^{-1}(w_n))$.

A pooling equilibrium exists in the n -type case if and only if

$$\sum_{i=1}^n P_i V_i > \max(g^{-1}(w_1), g^{-1}(w_2), \dots, g^{-1}(w_n)).$$

If this inequality holds, then

pooling at any common demand between the value of the left and right-hand sides would constitute a pure pooling equilibrium. The Bayesian defendant

would prefer settlement at that level to trial. Each type of plaintiff would prefer settlement at that level to trial. Therefore, it merely remains to find an $\alpha(d)$ function which would induce each plaintiff to prefer that common demand to any other real-valued demand. We proceed as we did in the 2-type case. Let d^* be the common demand level. Define

$$\alpha(d) \geq 0 \text{ for all } d, \text{ with equality at } d = V_n$$

$$\alpha(d) = 1 \text{ for } d \leq d^*$$

$$\text{and } \alpha(d) < \frac{g(d^*) - W_n}{g(d) - W_n} \text{ for } d > d^*.$$

By assumption, $\alpha(d) = 0$ for $d > V_n$. By arguments now familiar, this $\alpha(d)$ function will induce each type of plaintiff strictly to prefer d^* to any other real-valued demand.

The purpose of this section has been to show how the analysis and results of the 2-type case generalize. It should be noted, however, that other types of equilibria are also possible in the n -type case which have no counterpart in the 2-type case. Space limitations prevent a characterization of these other equilibria here.

VI. Conclusion

This paper has studied a model of pretrial settlement negotiations in which the prospective plaintiff makes a real-valued settlement demand of his choosing on a take-it-or-leave-it basis. Under complete information, the equilibrium of this game is known to be unique but never involves any litigation. In the more realistic cases of incomplete information, litigation does occur some fraction of the time even though there do in fact

exist pre-trial settlements which both parties would prefer to trial. If the plaintiff is uninformed, the defendant learns nothing from his demand. Bebchuk (1983) has studied this case and has shown under weak assumptions that its equilibrium is unique. We have studied the alternative case in which the plaintiff is the informed player. This is more plausible in personal injury cases. In such a case, defendant learning occurs in the equilibrium. However, the equilibrium ceases to be unique. The multiplicity of equilibria cannot be eliminated by the usual refinements of the Nash solution concept or by assuming that the injured plaintiff is drawn from one of a finite number or a continuum of possible types. A continuum of equilibria exists because the defendant's reception of demands which no plaintiff should make with positive probability is indeterminate.

Extending the model to the case where a single defendant--an insurance company--faces a sequence of plaintiffs of unknown type instead of facing only one plaintiff would be interesting. If plaintiffs were uncertain about some aspect of the insurance company and if the company's previous settlement behavior were observable, a "soft" company might then find it optimal to build a "reputation for toughness" by litigating frequently in early rounds. Such multi-stage analysis seems hopelessly intractable, however, in this case because of the multiplicity of the equilibria of each stage game.^{16/}

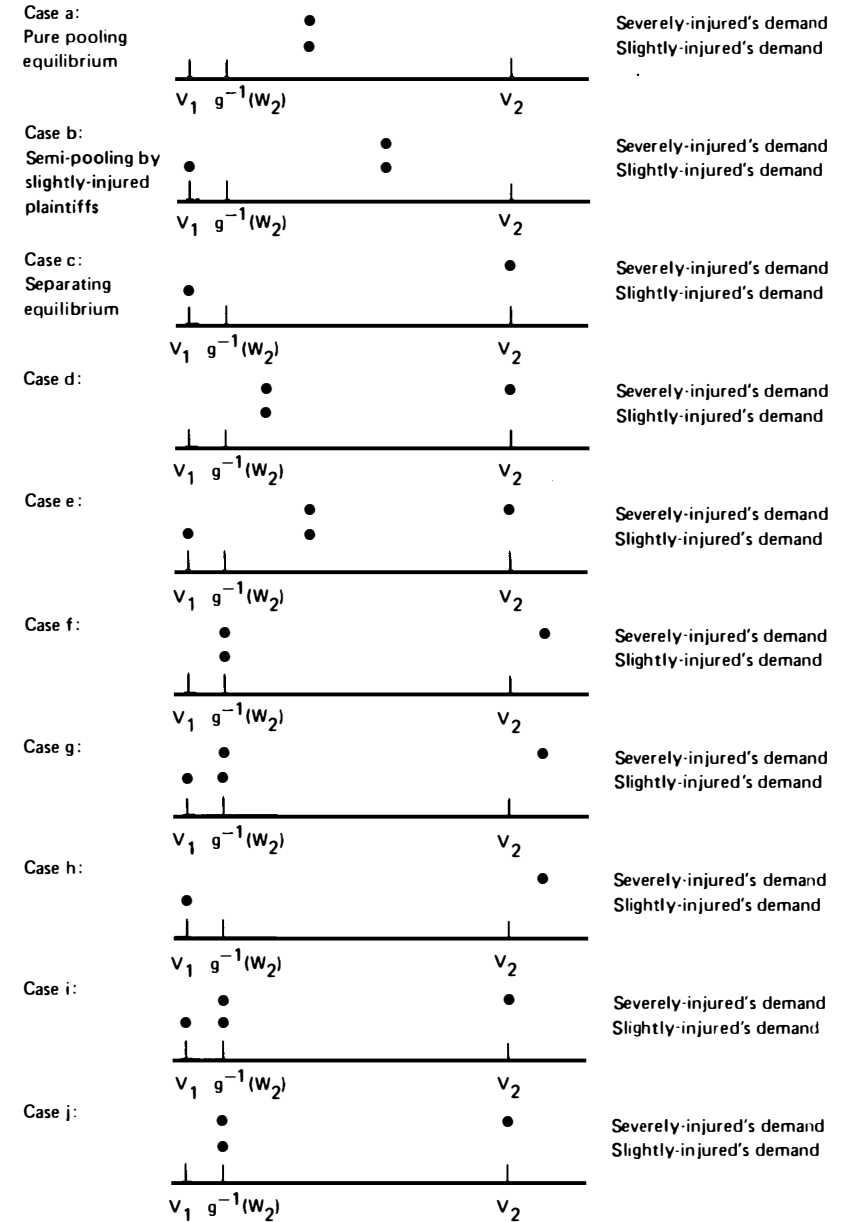


Fig. 1 — Location of jumps in plaintiffs' distribution functions supporting Nash equilibria

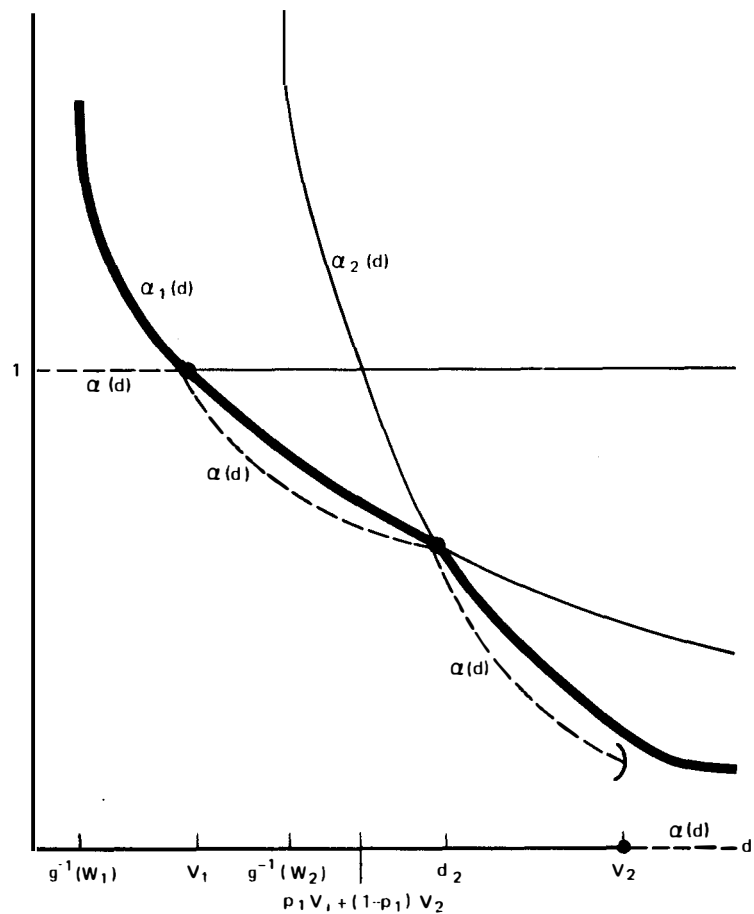


Fig. 2 – Defendant's strategy in a semi-pooling equilibrium

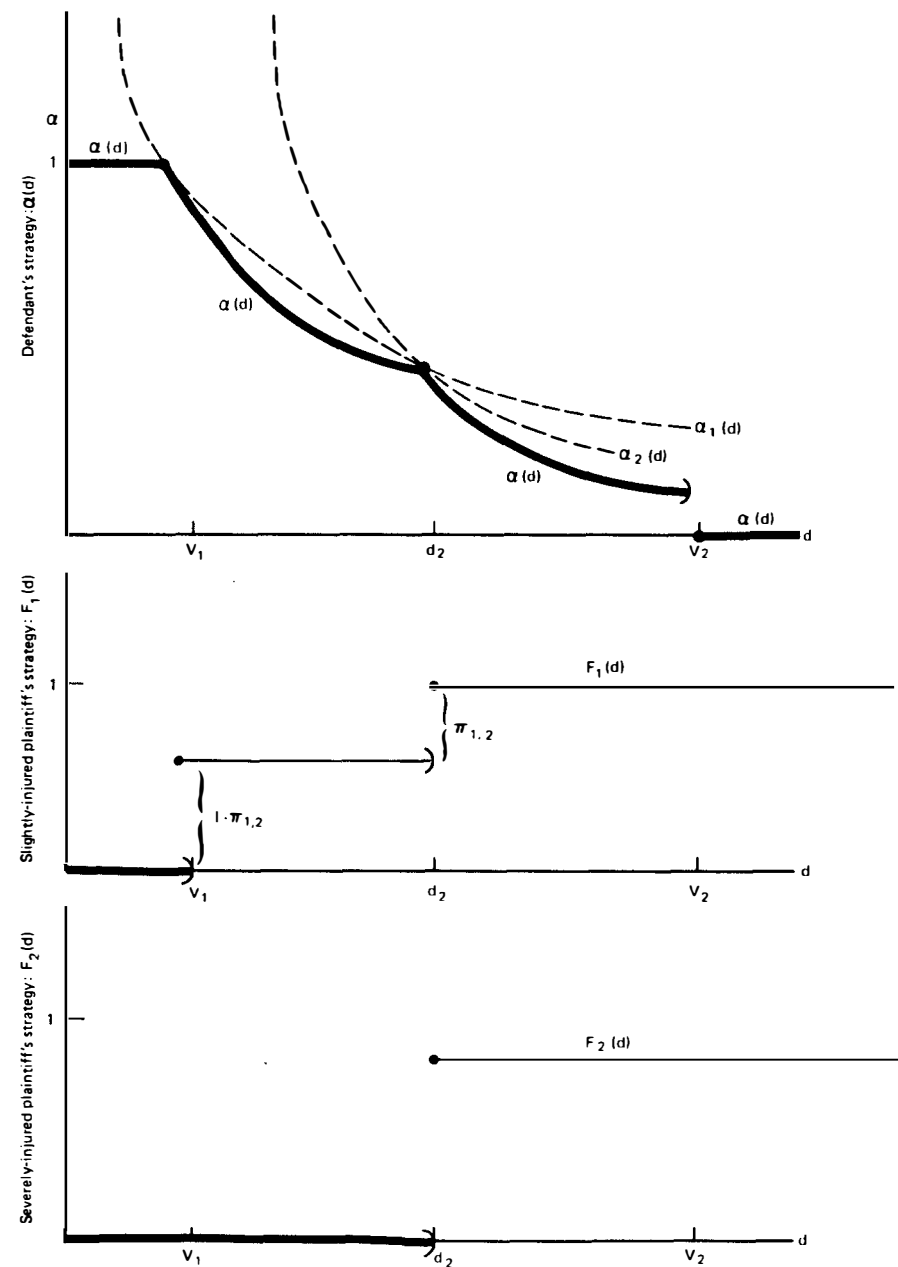


Fig. 3 – Strategies for each player in a semi-pooling equilibrium

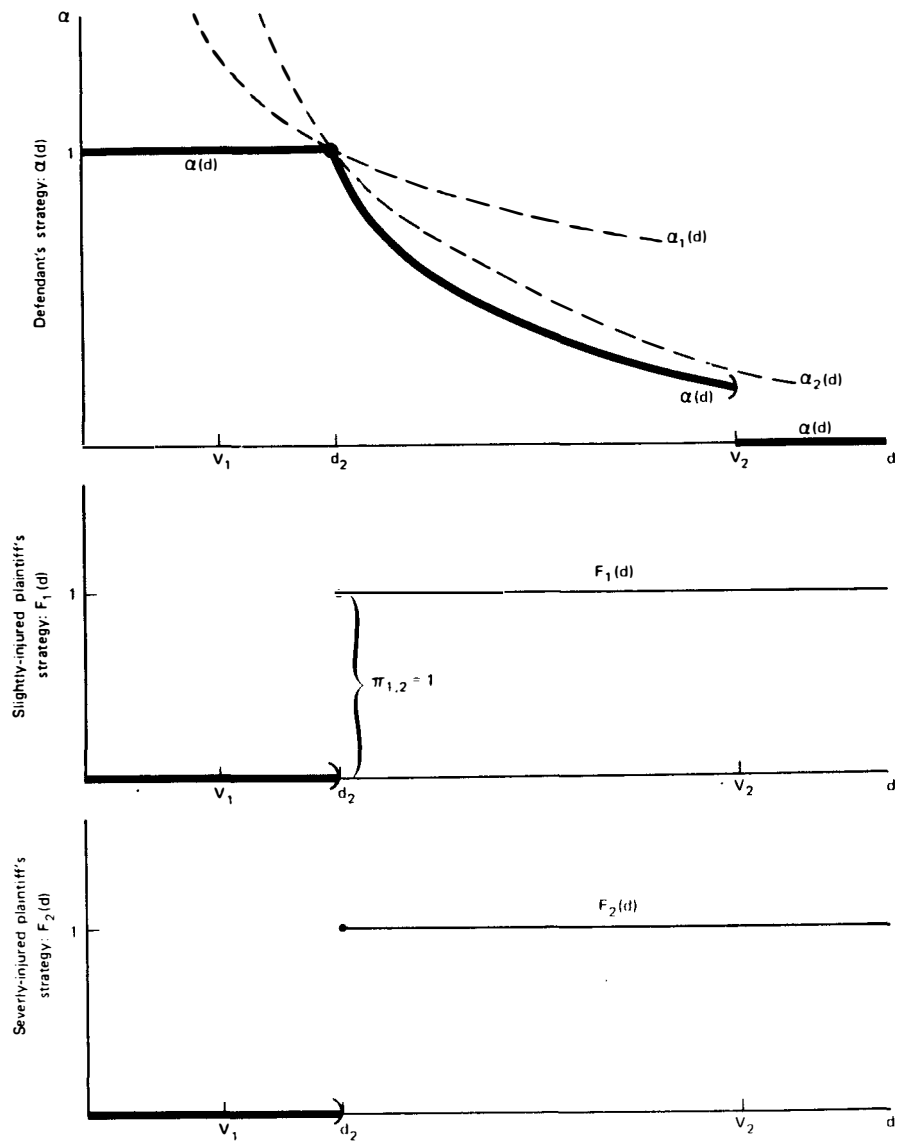


Fig. 4 – Strategies for each player in a pure pooling equilibrium

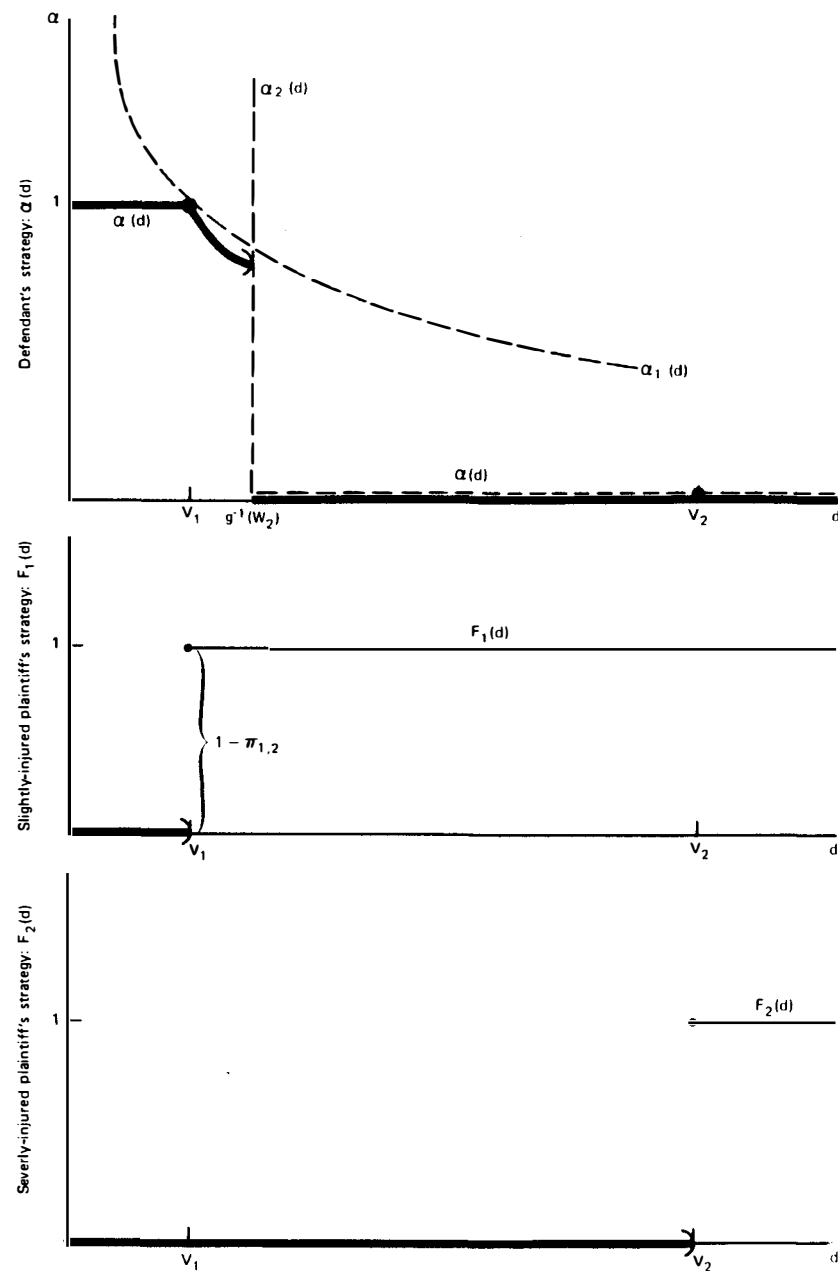
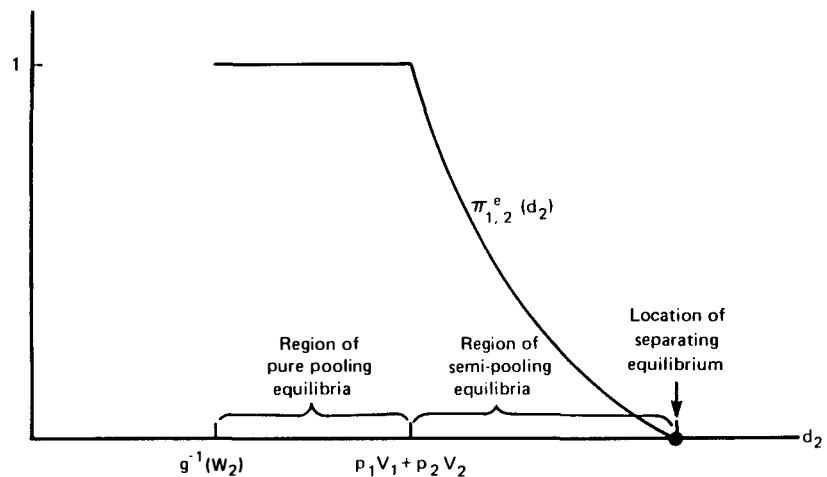
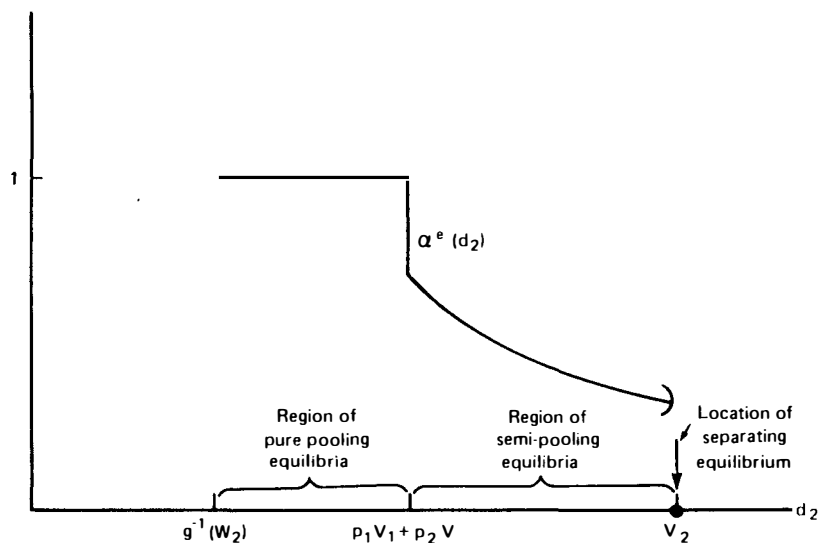


Fig. 5 – Strategies for each player in a separating equilibrium



Panel (a): Probability of an inflated claim in each of the possible equilibria



Panel (b): Probability a high claim is accepted in each of the possible equilibria

Fig. 6 – Summary characteristics of the strategies in each of the possible equilibria

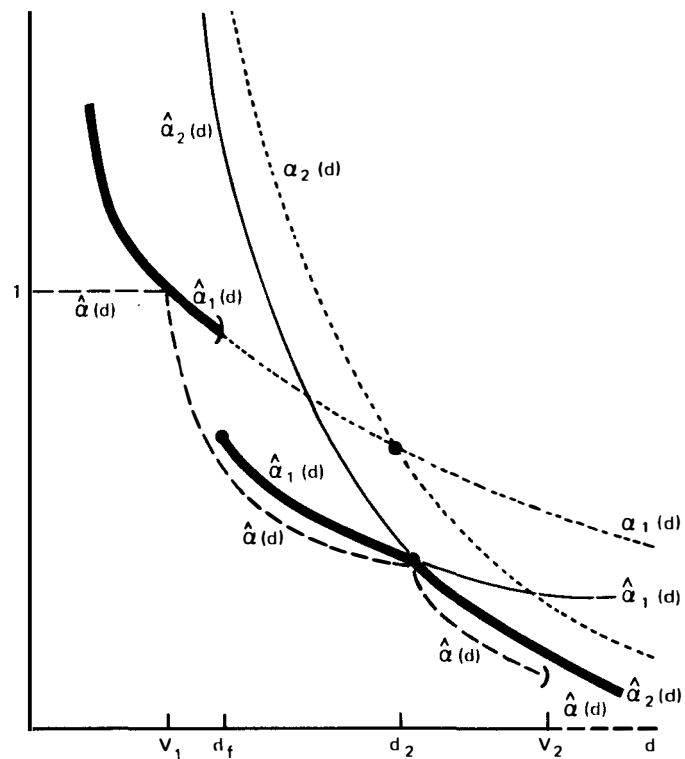


Fig. 7 – The effect of imposing a fee for filing claims below d_f on the defendant's strategy

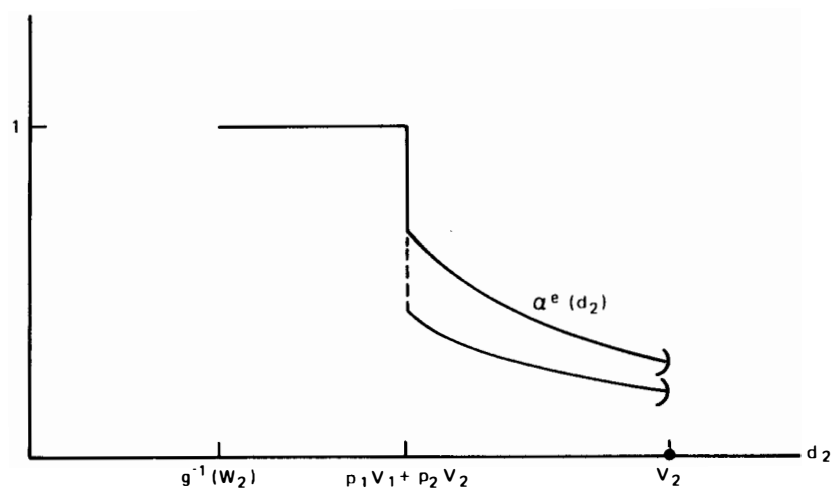


Fig. 8 – Reduced odds of acceptance of high settlement demands in response to fee for filing demands below d_f

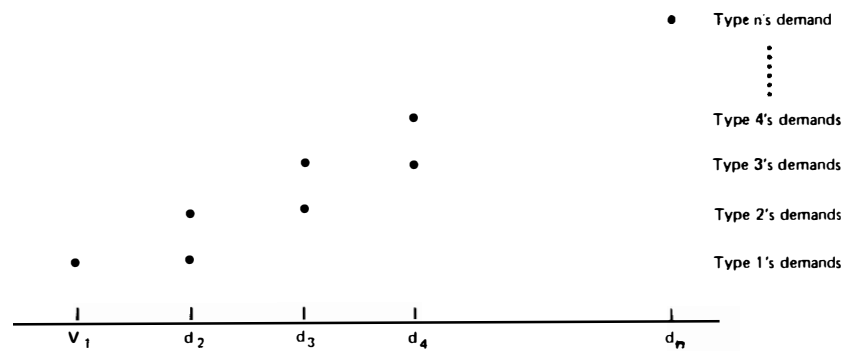


Fig. 9 – Location of jumps in plaintiffs' distribution functions in a semi-pooling equilibrium with n types

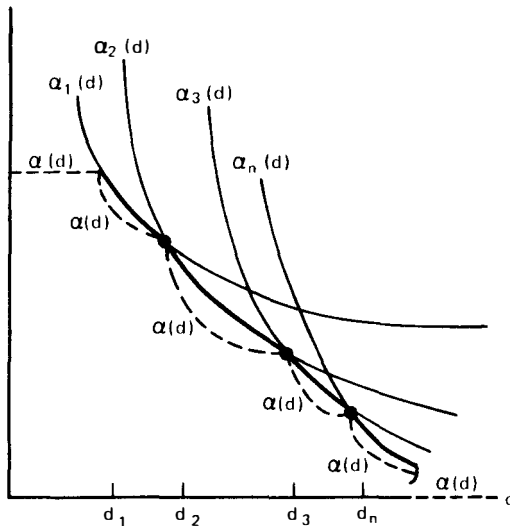


Fig. 10 – Defendant's strategy in a semi-pooling equilibrium with n types

*This paper extends the analysis in Salant and Rest [1982] by relaxing its restriction that plaintiffs must make one of two exogenous settlement demands. I would like to thank Gregory Rest for his collaboration in the earlier research effort. I would also like to express my deepest gratitude to Jonathan Cave for his many helpful suggestions.

- 1/ We use the Lebesgue-Stieltjes notation ($\int_{\beta} g(x) dF(x)$) here to mean either the sum $\sum_{x \in \beta} g(x) f(x)$ or the integral $\int_{\beta} g(x) f(x) dx$. In what follows, the equilibrium distribution functions (F_1) governing the demands of each plaintiff turn out to be step functions.
- 2/ The economist should recognize this as a familiar idea since he almost surely invokes it to explain the determination of aggregate output in an industry in which each firm has the same constant marginal cost schedule. In that case, aggregate output is determinate but the quantity produced by any individual firm at the equilibrium price is indeterminate.
- 3/ For example, to establish (b) assume that both plaintiffs did assign positive density to a common interval. Then the expected payoff of the severely-injured plaintiff would not change if instead he focussed the same probability mass on any low demand in the interval accepted with positive probability. Similarly the payoff of the slightly-injured plaintiff would not change if instead the probability mass he assigned to the entire interval were divided between the same low demand in the interval and some higher demand in the interval. But it is then obvious (from Proposition 3) that the severely-injured plaintiff could increase his expected payoff by putting all the probability mass he

assigned to the interval on the higher demand. Since the original strategy of the severely-injured plaintiff was not optimal, it would not occur in an equilibrium.

To establish (c), assume that the slightly-injured plaintiff did assign positive density to some interval and that the severely-injured plaintiff did assign positive probability to any point in that interval other than its upper boundary. Then the expected payoff of the slightly-injured plaintiff would not change if the same probability mass were instead allocated to two points--the demand also made by the severely-injured plaintiff and some higher demand in the interval accepted with positive probability. But it is then obvious (from Proposition 3) that the severely-injured plaintiff could increase his expected payoff by putting all the probability assigned to the interval on the higher demand. Since the original strategy of the severely-injured plaintiff was not optimal, it would not occur in an equilibrium. To establish the other part of (c), use same argument but reverse the roles of the two types of plaintiffs.

4/ In particular, if the slightly-injured plaintiff at least weakly prefers the higher of two demands, the severely-injured plaintiff must strictly prefer the higher demand.

5/ Use is made here of the simplification that the defendant is constrained to reject all demands exceeding V_2 . If instead the defendant were modeled as free to accept such high demands, a more intricate "properness" argument would be required to formalize our intuition that cases f-h are implausible.

6/ If the demand in the interval $(g^{-1}(w_2), v_2)$ was not accepted with positive probability, then no slightly-injured plaintiff would ever make it. But since in cases d and e, such plaintiffs are assumed to make the demand in this interval sometimes, that demand must be accepted with positive probability. But given this fact, we can infer that the V_2 demand must also be accepted with positive probability. Suppose the contrary. Then, every severely-injured plaintiff would strictly prefer to pool with the slightly-injured plaintiff at the lower demand since he prefers settlement at that level to trial and there is some chance of settlement. But this contradicts the assumption underlying cases d and e that the severely-injured plaintiff demands V_2 sometimes. Therefore, V_2 must be accepted with positive probability.

7/ Verification of i. and iii. are straightforward. To verify ii., note that

$$\frac{\alpha_1(d)}{\alpha_2(d)} = k \left[\frac{g(d) - w_2}{g(d) - w_1} \right]$$

$$\text{where } k = \left[\frac{g(v_1) - w_1}{g(d_2) - w_2} \right] \frac{1}{\alpha_1(d_2)} .$$

Since $\frac{g(d) - w_2}{g(d) - w_1}$ increases in d , $\frac{\alpha_1(d)}{\alpha_2(d)}$ must increase in d .

8/ The restrictions on $\alpha(d)$ are insufficient to determine $\alpha(d)$ uniquely. But since every candidate satisfying the restrictions would assign the same respective acceptance probabilities as $\alpha(d)$ to the two demands ($d = v_1$ and $d = d_2$) which plaintiffs make with positive

probability, this indeterminacy is of no consequence.

9/ To verify that an equilibrium is supported by undominated strategies, we must show that there exists no strategy for any player that is at least as good for that player as his equilibrium strategy against all feasible combinations of strategies for the other players. Hence, to show that a proposed strategy does not dominate the equilibrium strategy of some player, we only need to find one feasible combination of strategies for the other players which would make the proposed strategy inferior. Consider first the semi-pooling equilibrium. Since the severely-injured plaintiff strictly prefers d_2 to any other real-valued demand--given $\alpha(d)$ -- $F_2(d)$ cannot be dominated. Since the slightly-injured plaintiff strictly prefers d_2 or V_1 to any other real-valued demand--given $\alpha(d)$ --the only possibility for weakly dominating the equilibrium strategy is to give alternative weights to d_2 and V_1 . But if the proposed alternative were to give more weight to V_1 (resp. d_2), it would be inferior to $F_1(d)$ against an acceptance strategy of the defendant (some $\tilde{\alpha}(d)$) which makes the slightly-injured plaintiff strictly prefer demands of d_2 (resp. V_1) to demands of V_1 (resp. d_2). Hence $F_1(d)$ cannot be dominated. As for the defendant, we must show that no alternative ($\tilde{\alpha}(d)$) weakly dominates the equilibrium strategy $\alpha(d)$ against every feasible combination of strategies (F_1, F_2) of the plaintiffs. Suppose $\tilde{\alpha}(d') > \alpha(d')$ (resp. $\tilde{\alpha}(d') < \alpha(d')$) for some $d' \in (V_1, V_2)$. If F_1 (resp. F_2) jumped from zero to one at d' and F_2 (resp. F_1) jumped from zero to one at V_2 (resp. V_1) then the proposed strategy would be strictly inferior to the defendant's equilibrium strategy since the proposed alternative would cause demands of d' to be accepted more often (resp., less often). Consider next a

pure pooling equilibrium. This same argument can be used again to establish that $\alpha(d)$ is undominated. Moreover, $F_1(d)$ and $F_2(d)$ cannot be dominated since, in a pure pooling equilibrium, each plaintiff strictly prefers the common demand to any other--given $\alpha(d)$. Last, consider the separating equilibrium. Since the slightly-injured plaintiff strictly prefers to demand V_1 --given $\alpha(d)$ -- $F_1(d)$ cannot be dominated. Moreover, since any alternative to $F_2(d)$ is inferior for the severely-injured plaintiff if demands costing V_2 happened always to be accepted, $F_2(d)$ cannot be dominated. Finally, the argument used above establishes that the defendant's strategy ($\alpha(d)$) is undominated.

10/ Consider the defendant's conjectures in a semi-pooling equilibrium. Recall that his equilibrium strategy is as follows:

$$\begin{aligned} \alpha(d) &= 1 && \text{for } d \in [0, V_1] \\ \alpha(d) &\in (0, 1) && \text{for } d \in (V_1, V_2) \\ \alpha(d) &= 0 && \text{for } d = V_2. \end{aligned}$$

If this strategy is cost-minimizing for the defendant facing demand d , then his conjecture that he is facing a slightly-injured plaintiff ($C(d)$) must satisfy the following conditions:

$$\begin{aligned} d &\leq C(d)V_1 + (1 - C(d))V_2 && \text{for } d \in [0, V_1] \\ d &= C(d)V_1 + (1 - C(d))V_2 && \text{for } d \in (V_1, V_2) \\ d &\geq C(d)V_1 + (1 - C(d))V_2 && \text{for } d = V_2. \end{aligned}$$

$$\begin{aligned} \text{Hence } C(d) &\in [0, 1] && \text{for } d \in [0, v_1] \\ C(d) &= \frac{v_2 - d}{v_2 - v_1} \in (0, 1) && \text{for } d \in (v_1, v_2) \\ C(d) &\in [0, 1] && \text{for } d = v_2. \end{aligned}$$

Kreps-Wilson require that these conjectures be consistent with the defendant's initial beliefs (that slight injuries occur with probability p_1), with any information he may have observed (d) or inferred and, whenever possible, with the hypothesis that play has evolved to this point under the equilibrium strategies (severely-injured plaintiffs always demand d_2 while slightly-injured plaintiffs demand v_1 with probability $\pi_{1,2}$ and d_2 with complementary probability). That is, for those demands (v_1 and d_2) which occur in the equilibrium with positive probability, the conjecture must be consistent with Bayes' theorem. From Bayes' theorem,

$$\Pr(S_1 | d_2) = \frac{p_1 \pi_{1,2}}{p_1 \pi_{1,2} + (1 - p_1) \cdot 1}.$$

Recall that $\pi_{1,2} = \left(\frac{1 - p_1}{p_1} \right) \left(\frac{v_2 - d_2}{d_2 - v_1} \right).$

$$\text{Hence } \Pr(S_1 | d_2) = \frac{v_2 - d_2}{v_2 - v_1} = C(d_2).$$

$$\text{Similarly } \Pr(S_1 | v_1) = \frac{p_1 (1 - \pi_{1,2})}{p_1 (1 - \pi_{1,2}) + (1 - p_1) \cdot 0} = 1.$$

Hence we need merely restrict $C(d)$ so that $C(v_1) = 1$.

Any conjecture function of the following form would therefore be satisfactory:

$$\begin{aligned} C(d) &\in [0, 1] && \text{for } d \in [0, v_1) \\ C(d) &= 1 && \text{for } d = v_1 \\ C(d) &= \frac{v_2 - d}{v_2 - v_1} && \text{for } d \in (v_1, v_2) \\ C(d) &\in [0, 1] && \text{for } d = v_2. \end{aligned}$$

Some authors have proposed requiring that the conjecture function be continuous or monotone. For any semi-pooling equilibrium there exist $C(d)$ functions which simultaneously satisfy these additional restrictions. Hence any of the continuum of semi-pooling equilibria is sequential. A similar demonstration could be made for the pooling or separating equilibria.

11/ Underlying the defendant's equilibrium strategy ($\alpha(d)$) is his conjecture ($C(d)$) that the plaintiff making observed demand d is slightly injured. Even if demand d occurs with zero probability in equilibrium, Selten's perturbed-game approach in some cases permits the elimination of some conjectures as implausible. Although it could be used to rule out the equilibria we have already eliminated by other means, Selten's approach unfortunately permits us to rationalize all of the remaining equilibria. Imagine the two types of plaintiffs making a suboptimal demand d very infrequently. If the relative frequency of the errors of the two types of plaintiffs at each suboptimal d is set appropriately, then $C(d)$ can be derived by Bayes' theorem. As we let the two frequencies of errors by the two types of plaintiffs tend to zero while maintaining their ratio, we obtain in the limit our equilibrium--in which such errors happen with zero probability but defendants hold conjectures $C(d)$.

For details, see Cave [1984].

- 12/ That a continuum of equilibria exists in the finite-type case is evident from the discussion in Section V. A continuum of equilibria may also exist in the continuous-type case. Let the types be indexed by $i \in [0, 1]$ where a higher i reflects a more serious injury. Let $P(i)$ be the density function for the continuous random variable i . Assume it is common knowledge that plaintiffs are "drawn from" $P(i)$. Let $V(i)$ be the cost to the defendant of going to trial against a type- i plaintiff. $V(i)$ is strictly increasing. Suppose, given the exogenous data, that the following inequality holds:

$$\int_0^1 V(i)P(i)di > g^{-1}(V(1)) .$$

Then there will exist values which lie strictly between the left and right-hand sides of the inequality. Pick one such value and denote it d^* . If every type of plaintiff pooled at d^* , then defendants could infer nothing additional about the type of plaintiff by observing his demand. In that case, the expected cost to the defendant (the left-hand side above) would exceed the cost of settling (d^*). Hence, defendants would strictly prefer to settle at d^* . Moreover, since settling at d^* would be preferred to trial by the most seriously-injured plaintiff ($i = 1$), it would be preferred to trial by any of the less injured plaintiffs. Suppose the plaintiffs faced the following acceptance function:

$\alpha(d) \geq 0$ for all d , with equality at $d \geq V(1)$

$\alpha(d) = 1$ for $d \leq d^*$

$\alpha(d) < \frac{g(d^*) - W(1)}{g(d) - W(1)}$ for $d > d^*$.

Then every type of plaintiff would strictly prefer to pool at d^* . Since a continuum of possible values exist for d^* , a continuum of pooling equilibria exist.

- 13/ Bebchuk [1983] examines a legal settlements game with precisely the same sequence of moves but assumes that the first mover is uninformed. Multiple equilibria in such a case are exceptional and can easily be ruled out by weak assumptions.
- 14/ For simplicity, we assume that $g^{-1}(W_2) < p_1V_1 + p_2V_2$ so that a pooling region exists. It is trivial to adapt the analysis when the inequality is reversed and hence no such region exists.
- 15/ For simplicity, assume $f < g(V_1)$. That is, the fee is not so large that demands of V_1 would be unprofitable for plaintiffs.
- 16/ Ordover-Rubinstein (1983) analyze a sequential litigation game under incomplete information but fix the settlements which can be reached out of court exogenously. If such an approach is regarded as acceptable, it can also be used to extend the personal injury game of this paper to a multi-plaintiff setting. For a personal injury game where the settlement demand is restricted exogenously and the resulting equilibrium is unique, see Salant-Rest (1982).

REFERENCES

- Arrow, K., "The Property Rights Doctrine and Demand Revelation Under Incomplete Information," in M. Boskin (ed.), Essays in Honor of Tibor Scitovsky, Academic Press, New York, 1979.
- Bebchuk, L., "Litigation and Settlement Under Imperfect Information," Law and Economics Working Paper No. 15, Stanford Law School (November, 1983).
- Brazil, Wayne D., "The Adversary Character of Civil Discovery: A Critique and Proposals for Change," Vanderbilt Law Review, November 1978.
- Cave, J., "Perfect Equilibrium in Continuous Games," unpublished manuscript, January, 1984.
- Coase, R., "The Problem of Social Cost," Journal of Law and Economics, October 1960.
- Cooter, R., S. Marks and R. Mnookin, "Bargaining in the Shadow of the Law," Journal of Legal Studies, June 1982.
- Gould, J. P., "The Economics of Legal Conflicts," Journal of Legal Studies, June 1973.
- Harsanyi, J. C., "Games with Incomplete Information Played by 'Bayesian Players,'" Management Science, Part I in 1967 and Parts II and III in 1968.
- Landes, W. M., "An Economic Analysis of the Courts," Journal of Law and Economics, April 1971.
- Landes, W. M., and R. A. Posner, "Legal Precedent: A Theoretical and Empirical Analysis," Journal of Law and Economics, August 1976.
- Ordover, J., and A. Rubinstein, "A Pre-Trial Concession Game," New York University Working Paper (November 1983).
- P'ng, I., "Strategic Behavior in Suit, Settlement, and Trial," Bell Journal, Autumn, 1983.
- Posner, R. A., Economic Analysis of Law, Second Edition, Little, Brown, and Toronto, 1977.

- Rothschild, M., and J. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," Quarterly Journal of Economics 90, 1976, pp. 628-650.
- Salant, S., and G. Rest, "Litigation of Questioned Settlement Claims: A Bayesian Nash-Equilibrium Approach," The Rand Corporation, P-6809, September 1982.
- Shavell, S., "Suit, Settlement, and Trial: A Theoretical Analysis Under Alternative Methods for the Allocation of Legal Costs," Journal of Legal Studies, January 1982.